A Quantitative Approach to Detect Market Abuses: The Surveillance Automatic Integrated System (SAIS)

Market Abuse Phenomenon and Supervisory Authority

The problem for the Supervisory Authorities is:

The real-time identification of market abuse phenomena

Market Abuse Detection

Market Abuse Detection and Failure

...that’s because...

Actions which may be attributed to market abuse phenomena

Particular occurrence/eve not referred to a specific stock

How to detect a failure?

Through the exam of the elementary components which mainly affect the pattern of a stock and which characterise the trades made by the intermediaries

Market Abuse Phenomenon: definition

Market Abuses

Insider Trading

Manipulation

Market Based Manipulation

Information Based Manipulation
Marcello Minenna

Market Abuse Detection and Failure

The elementary components:

- Quantities
- Prices

Which may be realised employing diffusive processes.

The presence of abnormal returns is disclosed through an estimation of the returns auto-regressive models in discrete time.

The trading prices have to be the logarithm of the price; through the study of the dynamics of returns; the momentum effect components of the capture both the mean reversion and insider information is disclosed) or sharp changes (for example at moment

The elementary components in order to detect a Failure?

The Financial Literature

The Supervisory Experience

Quantitative Models

The quantitative methods for the abuse detection

Prices

The Financial Literature

The Supervisory Experience

- The trading prices have to be analysed in terms of returns, through the study of the dynamics of the logarithm of the price;
- Auto-regressive models in discrete time capture both the mean reversion and the momentum effect components of the returns;
- The presence of abnormal returns is disclosed through an estimation of the returns which may be realised employing diffusive processes.

The elementary components in order to detect a Failure?

The Financial Literature

The Supervisory Experience

Quantitative Models

The quantitative methods for the abuse detection

Quantities

- The quantities traded by each intermediary are examined in an aggregate way in terms of daily trading volumes according to an auto-regressive scheme;
- The market composition is assessed through two levels of analysis:
  1. The level of concentration of the intermediaries, that is the number of intermediaries and their shares in terms of trading volumes (so-called static concentration);
  2. The evolution of the concentration of the intermediaries, that is the change of each intermediary’s share in terms of trading volumes on a given security (so-called dynamic concentration).

...Hence, a Market Abuse Detection Procedure...

...requires the control of 4 financial variables:

- Prices
- Volumes
- Static Concentration
- Dynamic Concentration

...in order to implement models with predictive capabilities which allow the identification of abnormal movements in the variable examined (so-called Alerts)
The quantitative methods for the detection

**Financial Variable** ↔ **Reference Model**

**Alert**

### Automatic Integrated System for Markets Supervision

**The S.A.I.Vi.M: the procedure for the Market Abuse Detection**

**Implementation**

- Construction of the tripwires in order to detect abnormal movements of the financial variables: so-called **Alert**
- Definition of the algorithm that, by reading the alerts, identifies on a daily basis the stocks which entail some Failures, so-called **Warning**
- Comprehension of the reasons underlying the Warning and consequent decisions

### The S.A.I.Vi.M: the procedure for the Market Abuse Detection

- The stocks listed on the Market are different as regards:
  - Liquidity
  - Sector to which they belong
  - P/E

- The market is characterised by moments of boost/euphory or of crisis which may be generalized or bounded to some sectors (for instance, the 2000 bubble on technology stocks)

- The time horizon for the Failures analysis cannot be too long (for instance: a quarter, a semester, a year) in order to avoid the risk of losing sensitivity;
  - Changes in the strategic area of business of the company;
  - New listings

- The construction of the tripwires and of the algorithm which produces the Warning needs to be valid over all the stocks and to preserve the adequacy of its performance over time

### The joint reading of the results of the various alerts identifies the stocks for which there is a Failure, which becomes the Warning for the Consob

### The S.A.I.Vi.M: functioning

- For each stock listed on the Market
  - < 3 alert
  - 3 alert

### Enforcement Actions

- Tools of 2nd level
  - Yes
  - No

### The choice of the Models

- Developing the models for the tripwires through the employment of diffusive processes: that’s because diffusive processes exploiting some results of the stochastic limit theory prove to be:
  - Extremely suitable/proper for the representation of the phenomena
  - Good-performing even when the number of the observations is low
  - Able to simplify the problems concerning the estimation and the stability of the parameters
The set of stocks and of the relative observation periods has been selected by looking at those cases for which both the Failures and their reasons were known.

The Stocks Selection (n. 26) was oriented by:
- The presence of an investigation carried on by Consob;
- The existence of a Consob signalling to the Judicial Authority regarding an hypothesis of market abuse;
- The Liquidity of the stock;
- The Historical Volatility of the Stock;
- The price/earning ratio of the stock;
- The diffusion/spreading of the stock on the market.

The Calibration of the Procedure

The reference sample: The Stocks Selection (n.26) was oriented by:

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The period of the investigation
- The moment in which the stock was listed
- The operations of M&A
- The moment of the stock de-listing

The quantitative methods for the detection

The S.A.I.Vi.M. and the Empirical evidence:

Main Results

All the trading periods highlighted as critical in the reports for the Commission, since related to market abuse phenomena, have been detected.

Moreover have been highlighted other periods characterised by one of the following situations:
- The presence of rumors on the market, that is of news having the potential to be price sensitive;
- The existence of considerable changes in the movements of the financial variables analysed.
Empirical Evidence: Some Numbers

<table>
<thead>
<tr>
<th>Informational Reference of the Warning</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Report to the Commission</td>
<td>22%</td>
</tr>
<tr>
<td>Consob News</td>
<td>11%</td>
</tr>
<tr>
<td>Balance Sheet</td>
<td>10%</td>
</tr>
<tr>
<td>Information on the Net</td>
<td>53%</td>
</tr>
<tr>
<td>Data Analysis</td>
<td>4%</td>
</tr>
</tbody>
</table>

The construction of the alerts

The Price Alert

6 logical and computational steps

Employment of an auto-regressive model applied to the logarithmic transformation of the prices

The Logarithmic Transformation

\[ R_t = \log P_t \]

1) i. The Process in discrete time: AR(1)

\[ R_k = \alpha + \lambda R_{k-1} + \hat{\sigma} Z_k \]

\[ Z_k \sim N(0,1) \]

1) ii. The AR(1) Process in differential terms

by defining \( \lambda = 1-\gamma \) e \( \alpha = \gamma \cdot \eta \)

\[ R_k - R_{k-1} = \gamma \cdot (\eta - R_{k-1}) + \hat{\sigma} Z_k \]
2) The Re-Scaling of the Process:
The intervals are divided into \( \frac{1}{h} \) subintervals with a length \( h \):
\[
R_{kh} - R_{(k-1)h} = \gamma_h (\eta_h - R_{(k-1)h}) + \sigma \sqrt{h} Z_{kh}
\]

OR
\[
R_{kh} - R_{(k-1)h} = \gamma_h (\eta_h - R_{(k-1)h}) + \sigma Z_{kh}
\]

\( Z_{kh} \sim N(0, \sqrt{h}) \)

3) The weak convergence for \( h \downarrow 0 \):
\[
R_{kh} - R_{(k-1)h} = \gamma_h (\eta_h - R_{(k-1)h}) + \sigma R_{kh}
\]

\[
\lim_{h \downarrow 0} \frac{R_{kh} - R_{(k-1)h}}{h} = \gamma_h (\eta_h - R_{(k-1)h}) + \sigma R_{kh}
\]

\[
dR_i = q(\mu - R_i)dt + \sigma dW_i
\]

4) The SDE Properties
(Ornstein-Uhlenbeck Arithmetic process)
\[
R_t \sim N\left((R_{t-1} - \mu) e^{-\beta t} + \mu; \sqrt{\frac{\sigma^2}{2 \beta}} (1 - e^{-2\beta t})\right)
\]

5) The relation discrete versus continuous and the estimation of the parameters
(i) The specification of the AR (1) process allows to avoid useless numerical procedures
\[
R_k - R_{k-1} = \gamma (\eta_k - R_{k-1}) + \hat{\sigma} Z_k
\]

Employment of the SDE properties
\[
dR_t = q(\mu - R_t)dt + \sigma dW_t
\]

(ii) The relation discrete versus continuous time and the estimation of the parameters
(Imposing the equality between the first and the second conditional moments)
\[
R_k - R_{k-1} = (1-e^{-\beta}) \cdot \mu + (e^{-\beta} - 1) \cdot R_{k-1} + \sqrt{\frac{\sigma^2}{2 \beta}} (1-e^{-2\beta}) Z_k
\]

Regression Analysis
\[
R_k - R_{k-1} = \hat{a} + \hat{b} R_{k-1} + \hat{\epsilon}_k
\]

(iii) The relation discrete versus continuous time and the estimation of the parameters
\[
R_k - R_{k-1} = (1-e^{-\beta}) \cdot \mu + (e^{-\beta} - 1) \cdot R_{k-1} + \sqrt{\frac{\sigma^2}{2 \beta}} (1-e^{-2\beta}) Z_k
\]
The construction of the alerts

5) iv. The relation discrete versus continuous time and the estimation of the parameters

\[ \mu = -\frac{a}{b} \]
\[ q = \log(b+1)^{-1} \]
\[ \sigma = \sqrt{\sum_{i=1}^{n} \frac{\varepsilon_i^2}{n-2}} \cdot \sqrt{\frac{\log(b+1)^2}{b^2 + 2b}} \]

\[ k = 15 \quad \text{Infra-monthly analysis} \]

The Alert Activation

Example: The Price/Return alert

The models in discrete and in continuous time

\[ Q_t = \sum_i A(i) + V(i) \]

\[ A = \text{purchases} \]
\[ V = \text{sales} \]
\[ j \text{ denotes the intermediary} \]

The Volume Alert

6) The detection of the Abnormal pattern for the F.V:

**The normality Prediction Interval**

\[ P \left( \frac{\mu - z_{\alpha}}{\sigma} \sqrt{\frac{n}{2q}}(1 - e^{-2q}) + (R_i - \mu)e^{-q} \leq R_{i+1} \leq \mu + z_{\alpha} \sqrt{\frac{n}{2q}}(1 - e^{-2q}) + (R_i - \mu)e^{-q} \right) = \alpha \]

Raw Data examined according to an autocorrelation scheme

Infra-monthly analysis
The specified discrete process and the parameters estimation

\[ Q_k - Q_{k-1} = (e^{-\theta} - 1) \cdot Q_{k-1} + \frac{\sigma^2}{2\theta} (1 - e^{-2\theta}) Z_k \]

\[ \theta = \log(h+1) - 1 \]

\[ \sigma = \sqrt{\sum_{x} \frac{e_x^2}{n-1}} \cdot \sqrt{\log(h+1) + \frac{\sigma^2}{2\theta}} \]

\[ k = 15 \quad \text{INFRA-MONTHLY ANALYSIS} \]

The normality Prediction Interval

\[ P \left\{ \frac{z_{\alpha/2} \sqrt{\frac{\sigma^2}{2\theta} (1 - e^{-2\theta}) + Q e^{-\theta}}}{\mu + z_{\alpha/2} \sqrt{\frac{\sigma^2}{2\theta} (1 - e^{-2\theta}) + Q e^{-\theta}}} \leq \frac{Q_{i+1} \leq \mu + z_{\alpha/2} \sqrt{\frac{\sigma^2}{2\theta} (1 - e^{-2\theta}) + Q e^{-\theta}}} \right\} = \alpha \]

The Alerts on the Concentration

Definition of a synthetic indicator

Data examined according to a autocorrelation scheme

See technical note

Static Concentration

Consideration/Remark:

- The need to capture not only the movement in the variable for the total turnover of the market but also the possible directions taken by individual intermediaries and, hence the market, requires the definition of 3 different PRE-Alerts

\[ \hat{Q}_i = \sum_{i=1}^{n_i} \frac{\hat{Q}_i(i)}{\mu_i} \]

\[ \Theta_i = \frac{1}{n_i} \sum_{i=1}^{n_i} \left( \frac{\hat{Q}_i(i)}{\mu_i} \right) \]

WHERE

\[ \hat{Q}_i(i) = \sum_{i=1}^{n_i} Q_i(i) \]

\[ \mu_i = \frac{\sum_{i=1}^{n_i} Q_i(i)}{n_i} \]

\[ n_i \] is the number of intermediaries present on the market at time \( t \);

\[ Q_i(i), i = 1, \ldots, n_i \] are the quantities traded by the \( i \)-th intermediary at time \( t \).
The construction of the alerts

**Static Concentration**

See the technical note for the complete description of the mathematics on:

- The models in discrete and in continuous time
- The specified discrete process and the parameters estimation
- The normality prediction interval

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**Static Concentration**

The alert’s generation

\[ \Theta_i^A \rightarrow \text{At least one of the three pre-alerts produces 1 alert} \]

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**Static Concentration**

Consideration/Remark:

- Through some easy mathematical steps/passages it is possible to identify the intermediaries who generated the alert

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**Dynamic Concentration**

Dissimilarity index

\[ \Psi_i = \sqrt{\frac{1}{\bar{n}_i} \sum_{i=1}^{n_i} \tilde{Q}_i(i)^2} \]

Where:

\[ \tilde{Q}_i(i) = Q_i(i) - Q_{i-k}(i) \]

\[ \bar{n}_i = \sum_n \tilde{Q}_i(i) \neq 0 \]

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**Dynamic Concentration**

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- The need to capture not only the movement in the variable for the total turnover of the market but also the possible directions taken by individual intermediaries and, hence the market, requires the definition of 3 different pre-alerts

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**Dynamic Concentration**

The pre-alerts

Quantities bought

\[ Q_i^b = \sum_i A(i) \]

Net turnover

\[ Q_i = \sum_i A(i) - V(i) \]

Quantities sold

\[ Q_i^s = \sum_i V(i) \]
**Dynamic Concentration**

**See the technical note for the complete description of the Mathematics on:**

- The models in discrete and in continuous time
- The specified discrete process and the parameters estimation
- The normality prediction interval

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**Dynamic Concentration**

**The Alert’s Generation**

\[ \Psi_i \rightarrow \text{At least one of the} \]

\[ \Psi_i' \rightarrow \text{three pre-alerts produces} \]

\[ \Psi_i \rightarrow \text{1 alert} \]

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**Consideration/Remark:**

- Through some easy mathematical steps/passerages it is possible to identify/spot the intermediaries who generated the alert

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**Construction of the Algorithm for the Generation of the Warning**

\[ R_t \rightarrow \text{At least} \]

\[ Q_t \rightarrow \text{three of} \]

\[ \Theta_t \rightarrow \text{the four} \]

\[ \Psi_t \rightarrow \text{alerts generate the warning} \]

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**Market Abuse Phenomena and Supervisory Authority**

**The Software implementation of this procedure of Market Abuse detection represents:**

![CONSOB logo](CONSOB.png)

**Automatic Integrated System for Market Surveillance**